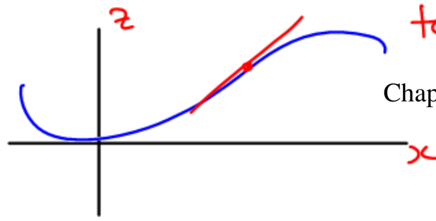
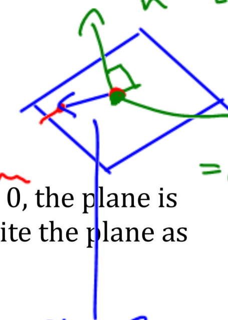


tangent line $z = z_0 + f'(a)(x-a)$



$\vec{n} = [a,b,c]$ known

$P_0 = [x_0, y_0, z_0]$

(x,y,z) on plane

5. Tangent planes

Recall that the equation

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

describes a plane passing through the point (x_0, y_0, z_0) with normal vector $[a, b, c]$. If $c = 0$, the plane is vertical (i.e. parallel to the z-axis) but otherwise we can divide by c and rearrange to write the plane as

$z = f(x,y)$ linear.

$$z = z_0 + A(x-x_0) + B(y-y_0)$$

where $A = -a/c$ and $B = -b/c$. This defines z as a linear function of x and y. Note that $\partial z/\partial x = A$ and $\partial z/\partial y = B$.

$0 = \vec{n} \cdot \vec{v} = [x-x_0, y-y_0, z-z_0]$

Definition. Given the graph of a function $f(x, y)$ (which is a surface in 3-space), and a point on the surface, the *tangent plane to the surface at that point* is the unique plane passing through that point that has the same slope as the surface in both the x- and the y-directions. Those two slopes are of course the two partial derivatives of the function at the point.

> The equation of the *tangent plane* to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is
>
> $$z = z_0 + \underbrace{f_x(x_0,y_0)}_{z/x \text{ slope}}(x-x_0) + \underbrace{f_y(x_0,y_0)}_{z/y \text{ slope}}(y-y_0)$$
>
> $$z = z_0 + \left.\frac{\partial z}{\partial x}\right|_{P_0}(x-x_0) + \left.\frac{\partial z}{\partial y}\right|_{P_0}(y-y_0)$$
>
> $P_0 = (x_0, y_0)$

together = tangent plane

= tangent line

Note that this plane has exactly the properties of the definition above––its slopes in the x and y direction are

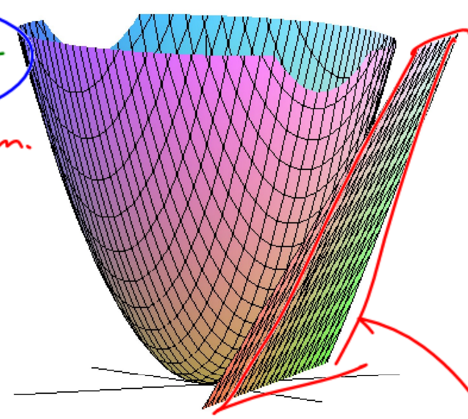
$$f_x(x_0,y_0) \text{ and } f_y(x_0,y_0)$$

Example 5.1. Find the tangent plane to the elliptic paraboloid

$$3z = x^2 + 2y^2 \to z = \frac{1}{3}x^2 + \frac{2}{3}y^2$$

$z = f(x,y)$ form.

at the point (2, 1, 2).
[Ans: $z-2 = (4/3)(x-2) + (4/3)(y-1)$.]



@ $(2,1,2) = (x_0, y_0, z_0)$

Need

$f_x = \frac{2}{3}x$

$f_y = \frac{4}{3}y$

$f_x = \frac{2}{3}(2) = 4/3$

$f_y = 4/3(1) = 4/3$

equality is a coincidence!

Tangent Plane $L(x,y) = 2 + \left(\frac{4}{3}\right)(x-2) + \left(\frac{4}{3}\right)(y-1)$

$z_0 + \underbrace{f_x}_{\text{slope}} \cdot (x-x_0) + \underbrace{f_y}_{\text{slope}} (y-y_0)$

L for linear

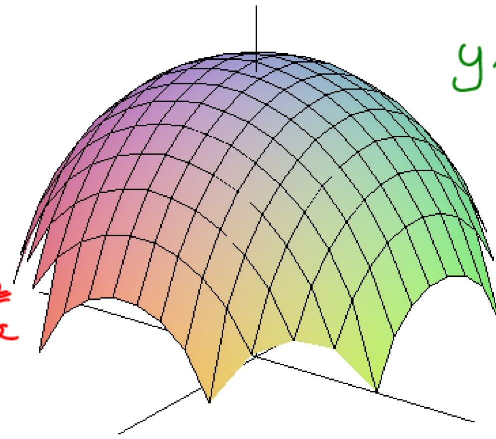
Example 5.2. (a) Use implicit differentiation to find the tangent plane to the sphere

(1) $x^2 + y^2 + z^2 = 9$ relation, not function

at the point (2, 1, 2).

[Ans: $z - 2 = -(x-2) - (y-1)/2$.]

$x \to z$
$y \to z$



Get $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ by implicit diff'n

Take $\frac{\partial}{\partial x}$ of both sides of (1)

$\frac{\partial}{\partial x} x = 1$

$\frac{\partial}{\partial x} y = 0$

$\frac{\partial}{\partial x} z = \frac{\partial z}{\partial x}$

$\partial/\partial x$ both sides.

$2x \cdot 1 + 0 + 2z \cdot \frac{\partial z}{\partial x} = 0$

Solve for $\boxed{\frac{\partial z}{\partial x} = \frac{-2x}{2z} = \frac{-x}{z}}$

Take $\frac{\partial}{\partial y}$ of both sides $\rightarrow$ $\partial/\partial y \, x = 0$, $\frac{\partial}{\partial y} y = 1$, $\frac{\partial}{\partial y} z = \frac{\partial z}{\partial y}$

$\vdots$

$\frac{\partial z}{\partial y} = -y/z$

x y z

Build plane @ (2,1,2)

@ (2,1,2)

$\frac{\partial z}{\partial x} = -x/z = -2/2$

and $\frac{\partial z}{\partial y} = -y/z = -1/2$

$z = L(x,y) = 2 + (-1)(x-2) + (-1/2)(y-1)$

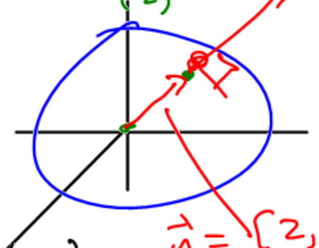
$z_0 + f_x (x - x_0) + f_y (y - y_0)$

@ point @ point.

(b) Use the geometry of the sphere to check your answer.

Yay! we can use our other plane formula to check (a) answer.

$\vec{n} = [2,1,2]$, $P_0 = [2,1,2]$



Plane: $2(x-2) + 1(y-1) + 2(z-2) = 0$ same plane?

$\vec{n} \cdot ((x,y,z) - (2,1,2)) = 0$

Isolate z:

$2(z-2) = -2(x-2) - 1(y-1)$

$z - 2 = -\frac{2}{2}(x-2) - \frac{1}{2}(y-1)$

$z = 2 + (-1)(x-2) + (-1/2)(y-1)$

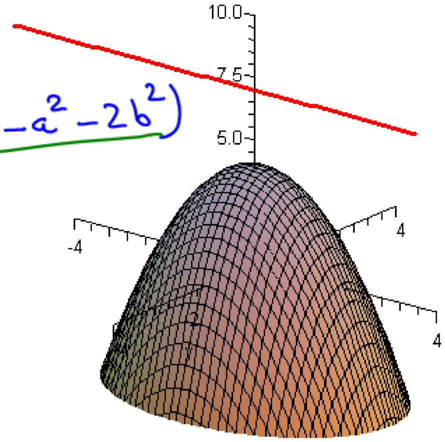
all the same!

Example 5.3. Find all tangent planes to the elliptic paraboloid

$$z = 4 - x^2 - 2y^2$$

that contain the line $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} + \begin{bmatrix} t \\ t \\ -2t \end{bmatrix}$.

$= (a, b, 4 - a^2 - 2b^2)$



[Ans: $z = 7 + 2x - 4y$, $z = 7 - 10x/3 + 4y/3$]

Build tangent planes at (a, b, c)

Need

$\frac{\partial z}{\partial x} = -2x$ @ (a,b) $\frac{\partial z}{\partial x} = -2a$

$\frac{\partial z}{\partial y} = -4y$ $\frac{\partial z}{\partial y} = -4b$

Tgt Plane: $z = L(x,y) = (4 - a^2 - 2b^2) + (-2a)(x - a) + (-4b)(y - b)$

$z_0 + f_x (x - x_0) + f_y (y - y_0)$

slope slope.

This plane must contain the line sub into plane

$x = t, \; y = t, \; z = 7 - 2t$

Must be true for all t's $\Rightarrow$ the whole line.

① $(7 - 2t) = (4 - a^2 - 2b^2) + (-2a)(t - a) + (-4b)(t - b)$

I.e. true for $t = 0$

① $\xrightarrow{t=0}$ $7 = 4 - a^2 - 2b^2 + 2a^2 + 4b^2$

or $3 = a^2 + 2b^2$ ②

also true for $t = 1$

① $\xrightarrow{t=1}$ $7 - 2 = 4 - a^2 - 2b^2 - 2a + 2a^2 - 4b + 4b^2$

$1 = (a^2 + 2b^2) - 2a - 4b$

so $1 = 3 - 2a - 4b \rightarrow 2a = 2 - 4b$ or $\boxed{a = 1 - 2b}$ ③

③ → ① $3 = (1 - 2b)^2 + 2b^2$

$$3 = (1 - 4b + 4b^2) + 2b^2$$

$$0 = -2 - 4b + 6b^2$$

tidy: /2 $\quad 0 = 3b^2 - 2b - 1$ $\quad$ $b=1$ is a root

factor $\quad 0 = (b-1)(3b+1)$

so $\quad b = 1 \xrightarrow[a=1-2b]{} a = 1-2 = -1$

and $\quad b = -1/3 \longrightarrow a = 1-2(-1/3) = 5/3$

so at $(a,b) = (-1,1)$ and $(5/3, -1/3)$

tgt planes include the given line.

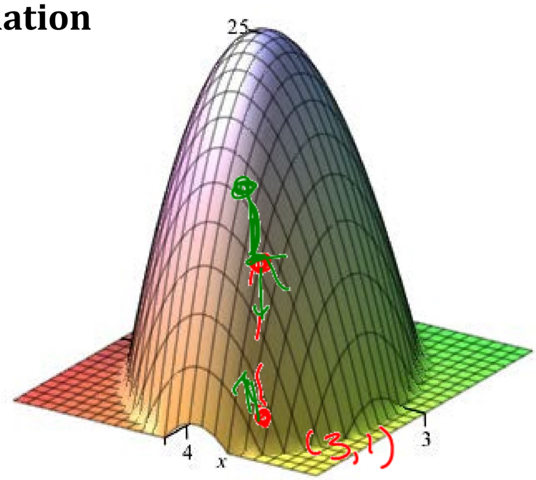
6. The multivariable chain rule and linear approximation

Example 6.1. Consider the paraboloid:

$$z = 25 - x^2 + xy - 4y^2$$

x_0, y_0 $\qquad z_0$

Suppose you are standing on the surface at the point $(\boxed{3, 1}, 15)$ and begin walking up the hill in the direction of the vector $[-2, -1]$. Find a general expression for the rate $\dot{z}$ at which you are ascending in terms of your x and y velocities.



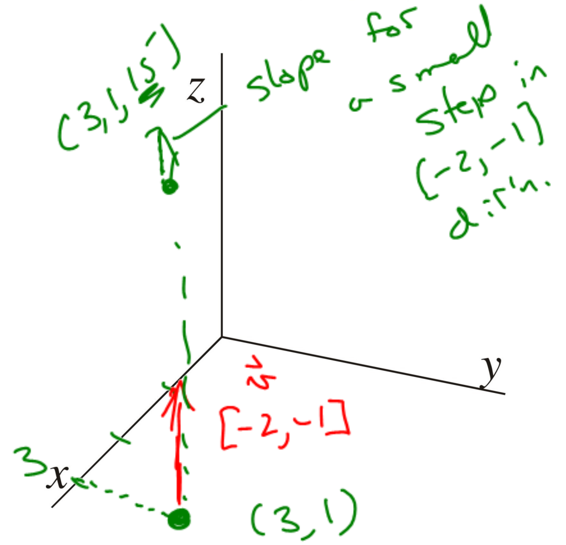
Need our trajectory as a parametric function

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + t \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

↓ easier form $\qquad$ deriv wrt time

$x = 3 - 2t \quad \Rightarrow \frac{d}{dt} \Rightarrow \quad \frac{dx}{dt} = \dot{x} = -2$

$y = 1 - t \quad \longrightarrow \quad \frac{dy}{dt} = \dot{y} = -1$



What about z?

$z = 25 - x^2 + xy - 4y^2$ (from above)

Take implicit deriv/ related rates
d/dt of both sides

$$\frac{d}{dt}(z) = \frac{d}{dt}\left(25 - x^2 + \underbrace{xy}_{\text{prod rule}} - 4y^2\right)$$

pattern search

$$\frac{dz}{dt} = -2x\frac{dx}{dt} + \left(\frac{dx}{dt}\cdot y + x\frac{dy}{dt}\right) - 8y\frac{dy}{dt}$$

$$\begin{tikzcd} & x \arrow[dr] & \\ t \arrow[ur] \arrow[dr] & & z \\ & y \arrow[ur] & \end{tikzcd}$$

Know $x = 3,\ y = 1,\ \frac{dx}{dt} = -2,\ \frac{dy}{dt} = -1$

$$\frac{dz}{dt} = -2(3)(-2) + (-2)(1) + (3)(-1) - 8(1)(-1)$$

$$= 12 - 2 - 3 + 8 = 15 \qquad (+)\text{ve} \Rightarrow \text{matches graph / uphill}$$

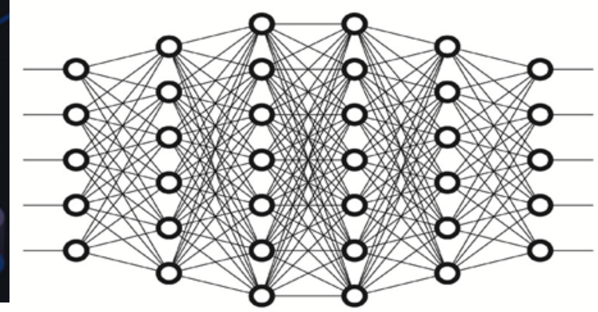
$$\frac{dz}{dt} = \underbrace{(-2x + y)}_{\frac{\partial z}{\partial x}}\frac{dx}{dt} + \underbrace{(x - 8y)}_{\frac{\partial z}{\partial y}}\frac{dy}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$d \to$ 1 input variable
$x(t)$
$y(t)$
$z(t)$
$\partial \to$ 2 or more inputs
$z(x, y)$

IBM Cloud

What is gradient descent?

Learn about gradient descent, an optimization algorithm used to train machine learning models by minimizing errors between predicted and actual results



Topology optimization

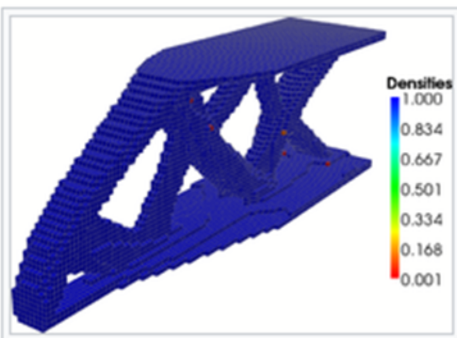
4 languages

Article Talk Read Edit View history

From Wikipedia, the free encyclopedia

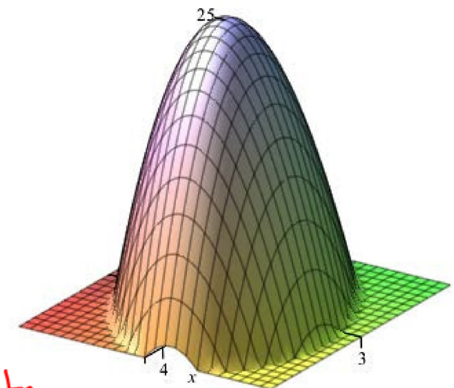
Topology optimization (TO) is a mathematical method that optimizes material layout within a given design space, for a given set of loads, boundary conditions and constraints with the goal of maximizing the performance of the system.
The conventional topology optimization formulation uses a finite element method (FEM) to evaluate the design performance. The design is optimized using either gradient-based mathematical programming techniques such as the optimality criteria algorithm and the method of moving asymptotes or non gradient-based algorithms such as genetic algorithms.

Topology optimization result when filtering is used



Topology optimization of a compliance problem

Sources:
- Wikipedia
- IBM Cloud



Example 6.2. The multivariable chain rule. In Example 6.1 you are walking on the hill

$$z = f(x, y)$$

and we found a general expression for the rate $\dot{z}$ at which you are ascending in terms of your x and y velocities. I'm going to write this in a whole bunch of equivalent ways—different notation and different ways of thinking about it.

GRADIENT vector.

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \left.\frac{\partial z}{\partial x}\right|_0 \frac{\mathrm{d}x}{\mathrm{d}t} + \left.\frac{\partial z}{\partial y}\right|_0 \frac{\mathrm{d}y}{\mathrm{d}t}$$

alternative way to write chain rule

new vector! package of all partial derivs of f(x,y)

$$\frac{\mathrm{d}}{\mathrm{d}t} f(x,y) = f_x|_0 \frac{\mathrm{d}x}{\mathrm{d}t} + f_y|_0 \frac{\mathrm{d}y}{\mathrm{d}t}$$

$$= [f_x \quad f_y]_0 \cdot [\dot{x} \quad \dot{y}]_0$$

velocity / xy direction vector.

These expressions describe instantaneous change at the base point (x_0, y_0) and the 0-subscript on the right-hand side emphasizes this. This is what is called *the multivariable chain rule*. It writes the rate of change of z as a sum of two terms. The first captures the component of change due to the fact that x is changing and the second captures the component of change due to the fact that y is changing. The formula says that we get the net rate of change by *adding* the two component one-variable changes. This is a significant result and it is not at first obvious.

The gradient of a function f of x and y at the point (x_0, y_0) is the vector of its partial derivatives:

GRADIENT symbol

$$\nabla f(x_0, y_0) = [f_x \quad f_y]$$

This concept gives us a natural way of writing the multivariable chain rule as a simple dot product.

But we shall soon see that the gradient has a number of powerful properties.

The last equations displays the fact that the right-hand side is a dot product and the vector of partial derivatives of f is called *the gradient of f*. It is written ∇f and is called *grad f*. So the last equation says that the rate of change of z is the dot product of the gradient of f and the velocity vector $\mathbf{v} = [\dot{x} \quad \dot{y}]$.

This writes the multivariable equation as a direct generalization of the one-variable chain rule for $z = f(x)$:

$$\frac{\mathrm{d}z}{\mathrm{d}t} = f'(x)\frac{\mathrm{d}x}{\mathrm{d}t}$$

v in xy

Example 6.3 A bug is walking up the parabola $y = x^2$ at speed 0.3. The temperature of the landscape depends on position according to the equation

$$T = x^{1.2} y^{0.8}.$$

→ at (0,0), at 0°, as x, y incr above 0 → T increases

At what rate is the bug experiencing the temperature change at $x = y = 1$?

want $\frac{dT}{dt}$

$\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\frac{dT}{dt} = (\nabla T)\cdot(\vec{v})$$

$$= [T_x, T_y]\cdot\left[\frac{dx}{dt}, \frac{dy}{dt}\right]$$

Find values $T_x = 1.2x^{0.2}y^{0.8}$

and $T_y = x^{1.2}(0.8y^{-0.2})$

at $x=1,\ y=1$

$T_x = (1.2)(1)(1) = 1.2$

$T_y = 0.8$

$\frac{d}{dx}x^3 = 3x^2$

$\frac{d}{dx}x^{1.2} = 1.2x^{0.2}$

To get $\vec{v}$, we can parameterize $y=x^2$ parabola

Let $x=t \rightarrow \frac{dx}{dt} = 1$

so $y = x^2 = t^2$ $\quad \frac{dy}{dt} = 2t$

at $(1,1) \to t=1$

$\frac{dx}{dt} = 1 \rightarrow \vec{v} = \begin{bmatrix}1\\2\end{bmatrix}$

$\underbrace{\frac{dy}{dt} = 2}_{\text{dir'n okay}}$

but speed is

$\|\vec{v}\| = \sqrt{1^2+2^2} = \sqrt{5}$

want ~~0.3~~

use $\boxed{\vec{v} = 0.3\Big(\underbrace{\frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}}_{\text{unit vector/ length 1 vector}}\Big)}$

$\frac{dT}{dt} = (\nabla T)\cdot\vec{v}$

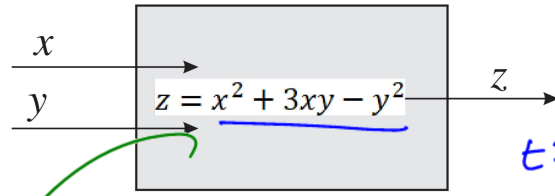
$= [1.2, 0.8]\cdot\left(0.3\cdot\frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}\right)$ $\quad$ °C/m $\quad$ m/s

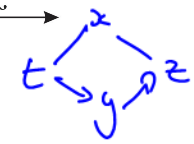
$= \frac{1}{\sqrt{5}}(0.84) \cong 0.38\ {}^\circ C/s$

$1/4 = 3/4$

Example 6.4. Let the output z be determined by two inputs x and y according to the formula:

$$z = f(x, y) = x^2 + 3xy - y^2$$





Suppose our base values are

$$x_0 = 25 \quad y_0 = 10$$
$$z_0 = f(25,10) = 1275$$

(a) Suppose that at the base point, the inputs are changing at rate $\dot{x} = 0.3$ and $\dot{y} = 0.2$. At what rate is z changing? [Answer: $\dot{z} = 35$]

$\frac{dx}{dt}$ $\frac{dy}{dt}$

Chain rule $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$ Know

Need... found!

$= (80)(0.3) + (55)(0.2)$

$\frac{dz}{dt} = 35 \ \frac{(\text{units of } z)}{\text{unit time}}$

$\frac{\partial z}{\partial x} = 2x + 3y$

$\frac{\partial z}{\partial y} = 3x - 2y$

@ $x_0 = 25,\ y_0 = 10$

$\boxed{\frac{\partial z}{\partial x}} = 50 + 30 = \boxed{80}$

$\boxed{\frac{\partial z}{\partial y}} = 75 - 20 = \boxed{55}$

(b) Find the equation of the tangent plane to the graph $z = f(x, y)$ at the base point (25, 10). [Answer: $z = 1275 + 80(x - 25) + 55(y - 10)$]

Tangent plane $z = z_0 + \frac{\partial z}{\partial x}(x - x_0) + \frac{\partial z}{\partial y}(y - y_0)$ tgt plane.

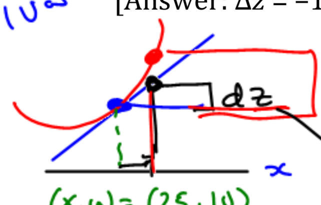
$= 1275 + 80(x - 25) + 55(y - 10)$

$\Delta x = -2$

(c) For the function defined above, suppose that x changes from 25 to 23 and y changes from 10 to 11. Calculate the real change Δz and the differential change dz.
[Answer: $\Delta z = -108$, $dz = -105$]

$\Delta y = +1$

1 variable



Δz = real, just $f(x,y)$

diff'l / target plane

real change

$\Delta z = f(23, 11) - f(25, 10)$

$= (23^2 - 3(23)(11) - 11^2) - 1275$

$= 1167 - 1275$

$= -108$

Compare to dz (tangent plane)

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$= (80)(-2) + (55)(+1)$

$\boxed{= -105.}$

Example 6.5. A function of three variables. The temperature at the point (x, y, z) is

$$T(x, y, z) = xe^{yz-x}$$

$$\frac{\partial T}{\partial x} = (1)e^{yz-x} + (x)e^{yz-x}(-1)$$

$$\frac{\partial T}{\partial y} = xe^{yz-x}(z)$$

$$\partial T/\partial z = xe^{yz-x}(y)$$

(a) A bug is at the point (2, 1, 2) and is flying through the air. Find an expression for the rate at which the temperature of the air around it changes in terms of the rates of change of its coordinates $\dot{x}$, $\dot{y}$ and $\dot{z}$. [Answer: $\dot{T} = -\dot{x} + 4\dot{y} + 2\dot{z}$]

$\frac{dT}{dt}$

@ (2, 1, 2)

$yz - x = 0$

so $e^{yz-x} = e^0 = 1$

so

$$\frac{\partial T}{\partial x} = 1 - 2 = -1$$

$$\frac{\partial T}{\partial y} = 2 \cdot 1 \cdot 2 = 4$$

$$\frac{\partial T}{\partial z} = 2 \cdot 1 \cdot 1 = 2$$

Chain rule
3 variable

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\underbrace{\frac{dx}{dt}}_{=\dot{x}} + \frac{\partial T}{\partial y}\underbrace{\frac{dy}{dt}}_{=\dot{y}} + \frac{\partial T}{\partial z}\underbrace{\frac{dz}{dt}}_{=\dot{z}}$$

know

$$\dot{T} = (-1)\dot{x} + (4)\dot{y} + 2\dot{z}$$

(b) Suppose the path of the bug is described by the equation

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t+2 \\ t^2+1 \\ 2-t \end{bmatrix}$$

@ t=0 $x = 2$, $y = 1$, $z = 2$ Same as @

$$\begin{tikzcd} & x \arrow[dr] & \\ t \arrow[ur] \arrow[r] \arrow[dr] & y \arrow[r] & T \\ & z \arrow[ur] & \end{tikzcd}$$

where t is time. Using the temperature function above, at what rate does it experience the temperature changing at $t = 0$?

[Answer: –3]

$\dot{x} = 1$

$\dot{y} = 2t$

$\dot{z} = -1$

@ t=0 $\dot{x} = 1$, $\dot{y} = 0$, $\dot{z} = -1$

$\vec{v} = [1, 0, -1]$

so $\dot{T}$ at $t = 0$, and (2, 1, 2) is

$$\dot{T} = (-1)(1) + (4)(0) + 2(-1)$$

$$= -3 \ {}^\circ C/s$$

(c) Suppose the bug in part (a) flies through the point (2, 1, 2) at speed 1 in the direction of the vector [1, 2, –1]. What is the rate at which the temperature of the air around it changes? [Answer: $\dot{T} = 5/\sqrt{6}$]

$\vec{v}$ direction, $\|[1, 2, -1]\| = \sqrt{6}$

want my $\|\vec{v}\| = 1$, $\vec{v} = \frac{1}{\sqrt{6}}[1, 2, -1]$

scaled to make length 1

same dir'n

$\dot{x} = 1/\sqrt{6}$

$\dot{y} = 2/\sqrt{6}$

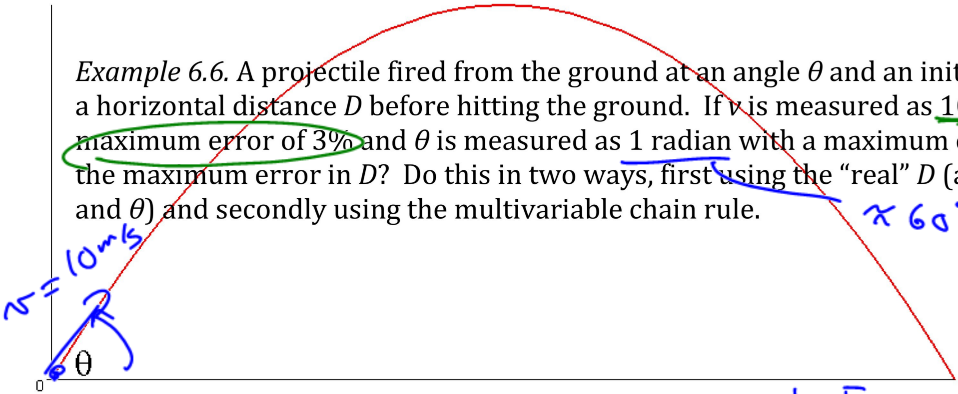
$\dot{z} = -1/\sqrt{6}$

$$\dot{T} = \nabla T \cdot \vec{v} = [-1, 4, 2] \cdot \left(\frac{1}{\sqrt{6}}[1, 2, -1]\right)$$

$$\underbrace{\left[\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right]}_{\text{gradient vector}} \cdot \left[\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right] = \frac{(-1 + 8 - 2)}{\sqrt{6}} = 5/\sqrt{6}$$

Example 6.6. A projectile fired from the ground at an angle θ and an initial velocity v travels a horizontal distance D before hitting the ground. If v is measured as 10 m/s with a maximum error of 3% and θ is measured as 1 radian with a maximum error of 2%, what is the maximum error in D? Do this in two ways, first using the "real" D (as a function of v and θ) and secondly using the multivariable chain rule.

$\approx 60°$



$$D(v,\theta) = \frac{v^2}{g}\sin(2\theta)$$

Eg. $D_0(\underset{m/s}{10}, \underset{rad}{1}) = \frac{100}{9.8}\sin(2) \cong \boxed{9.279\text{ m}}$

$v_0 = 10$ → $v_{max} = 10 + \underbrace{0.3}_{3\%\text{ of }10} = 10.3$ → $v_{min} = 10 - 0.3 = 9.7$

$\theta_0 = 1$ → $\theta_{max} = 1 + \underbrace{0.02}_{2\%} = 1.02$ → $\theta_{min} = 1 - 0.02 = 0.98$

Real D_{max} / most extreme larger D

$$D_{max}(\underset{10.3}{v_{max}}, \underset{0.98}{\theta_{min}}) = \frac{(10.3)^2\sin(2\cdot 0.98)}{9.8} \cong \boxed{10.016}$$

or $\boxed{\Delta D \cong 0.737}$ similar

and D_{min} / most extreme smaller D

$$D_{min}(\underset{9.7\text{ m/s}}{v_{min}}, \underset{1.02\text{ rad}}{\theta_{max}}) = \frac{(9.7)^2\sin(2\cdot 1.02)}{9.8} \cong \boxed{8.563\text{ m}}$$

or $\Delta D \cong 9.279 - 8.563$ $\boxed{\cong 0.716}$

Estimated error dD using linearization / tgt plane.

Start w/ $D(v,\theta) = \dfrac{v^2\sin(2\theta)}{g}$

Compute $\dfrac{\partial D}{\partial v} = \dfrac{2v\cdot\sin(2\theta)}{g}$ at $(10, 1)$ v_0, θ_0 $\dfrac{\partial D}{\partial v} = \dfrac{2(10)\sin(2)}{9.8} \cong 1.856$

and $\dfrac{\partial D}{\partial \theta} = \dfrac{v^2}{g}\cos(2\theta)\cdot 2$ | $\dfrac{\partial D}{\partial \theta} = \dfrac{(10)^2}{9.8}\cos(2)\cdot 2 = -8.493$

tgt plane/ linearization

Estimated $D = D_0 + \underbrace{(1.856)}_{\frac{\partial D}{\partial v}}\Delta v + \underbrace{(-8.493)}_{\frac{\partial D}{\partial \theta}}\Delta\theta$

$\Delta v = +0.3$ or -0.3

Extreme $D_{max} = 9.279 + (1.856)\cdot(+0.3) + (-8.493)\cdot(-0.02)$

m/s

$\Delta\theta = +0.02$ or -0.02

$= 10.006$ m

" except opp signs

Extreme $D_{min} = 9.279 + (1.856)(-0.3) + (-8.493)(+0.02)$

$= 8.552$

ΔD_1 " $D_{max} - D$ and ΔD_2 " $D - D_{min}$

Both ΔD's $= 0.727$