

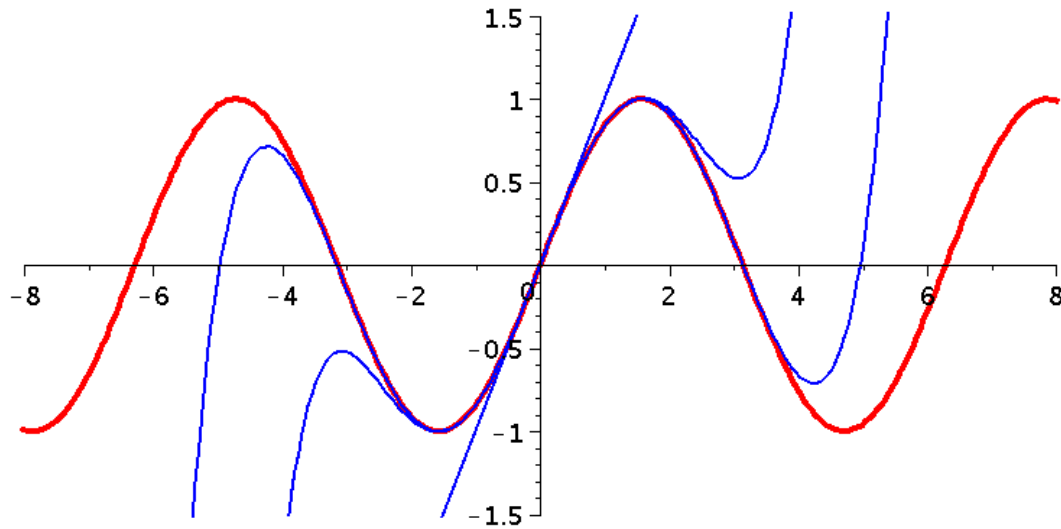
# Chapter 2

Short:

just

2 weeks

# Infinite Series and Power Series



1. Series: general concepts
2. Alternating series
3. Power series
4. Taylor and Maclaurin series
5. Approximating functions by polynomials
6. The great Leonhard Euler (1707 – 1783)

**1. Series: General concepts**

An infinite series is an infinite sum of the form:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

Define its nth partial sum  $s_n$  as the sum of the first  $n$  terms:

$$s_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

If the sequence  $\{s_n\}$  converges to a finite limit  $s$  as  $n$  approaches infinity, we say that the series converges, and that  $s$  is the sum of the series and write.

$$s = \sum_{n=1}^{\infty} a_k = \lim_{n \rightarrow \infty} s_n$$

If the series fails to converge, we say that it diverges.

*Example 1.1.* In each case does the series converge or diverge?

(a)  $\sum_{n=1}^{\infty} n = 1 + 2 + 3 + 4 + \dots$

*Handwritten notes:* indices  $n=1, n=2, n=3$  point to terms.  $s_1=1, s_2=1+2=3, s_3=1+2+3=6, s_4=1+2+3+4=10$ .  $\lim_{n \rightarrow \infty} s_n \rightarrow \infty$  or not finite.

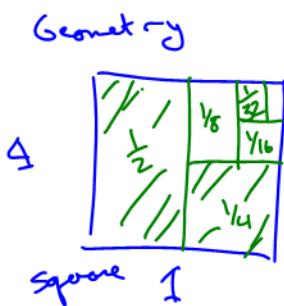
(b)  $\sum_{n=1}^{\infty} (-1)^{n-1} = 1 - 1 + 1 - 1 + 1 - \dots$

*Handwritten notes:* Partial sums  $s_1=1, s_2=0, s_3=1, s_4=0$ .  $\lim_{n \rightarrow \infty} s_n \rightarrow$  a single finite number? No!  $\Rightarrow$  this series also diverges. So this series diverges.

*Example 1.2.* Find the partial sums of the series:

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n} + \dots$$

Provide an intuitive argument that the series converges and find its sum.



*Intuition*

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$$

single, finite value

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{2^k}$$

This series converges!

*With formulas*

$n=1$	$s_1 = \frac{1}{2}$	$= 1 - \frac{1}{2}$	<i>Pattern</i>
$n=2$	$s_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$	$= 1 - \frac{1}{4} = 1 - \frac{1}{2^2}$	
$n=3$	$s_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$	$= 1 - \frac{1}{8}$	

$$n=4 \quad S_4 = \dots = \frac{15}{16} = 1 - \frac{1}{2^4}$$

⋮

for any  $n^{\text{th}}$  partial sum,

$$S_n = 1 - \frac{1}{2^n}$$

e.g. sum of the first 20 terms =  $S_{20}$   
 $= 1 - \frac{1}{2^{20}}$

The infinite sum

$$= \lim_{n \rightarrow \infty} S_n$$

$S_n$

↓ formula for  $S_n$

$$= \lim_{n \rightarrow \infty}$$

$n^{\text{th}}$  partial sum

$$1 - \left( \frac{1}{2^n} \right) = 1 - 0 = \boxed{1}$$

as  $n \rightarrow \infty$ ,  $\frac{1}{2^n} \rightarrow 0$

~~diverges~~  
 $r=2$

$$3 + 6 + 12 + 24 + 48 + \dots$$

eg.  $3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \dots$  converges  
 $r = 1/4$

Example 1.3 A series in which each term is obtained from the previous term by multiplication by a constant  $r$  is called *geometric*. If we denote the first term by  $a$ , the series has the form:

$$\sum_{k=0}^{\infty} ar^k = \sum_{n=1}^{\infty} ar^{n-1} = a + ar^1 + ar^2 + ar^3 + \dots$$

$a =$  first term  
 $r =$  multiplier/  
 ratio

Show that the series converges if and only if  $|r| < 1$ , and in this case its sum is  $s = \frac{a}{1-r}$ .

Consider the finite sum

$$S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} = \text{??} \text{ } n \text{ terms}$$

also consider same sum, just  $(r)$

$$r \cdot S_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$$

Subtract

$$S_n - rS_n = a + 0 + 0 + \dots + 0 - ar^n$$

tidy:  $S_n(1-r) = a - ar^n$  flip 1-r's,

Solve for the sum  $S_n$ :

$$\text{finite} \rightarrow S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$

↓ formula

we want

$$S = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r}$$

Two eg.  $2^n \rightarrow \infty$   
 $0.5^n \rightarrow 0$

As an example, find the sum of the geometric series.  $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$

- 1) Find  $a, r$ :  $a = 5$   $r = -2/3$
- 2) Determine if series converges:  $|r| = |-2/3| < 1$   
 This series converges
- 3) Find sum:  $S = \frac{a}{1-r} = \frac{5}{1-(-2/3)} = \frac{5}{5/3} = 3$

Note. Most of our work here will be with series that have a strong geometric-series character, that is they have the form

$$\sum_{n=0}^{\infty} a_n r^n = a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \dots$$

for given coefficients  $a_n$ , and this will enable us to decide whether they converge and to find their sum.

An interesting example is the arithmetic-geometric series:  $\sum_{n=1}^{\infty} nr^n = r + 2r^2 + 3r^3 + 4r^4 + \dots$

This series converges for  $|r| < 1$ . Can you find the sum?

$\lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r}$ 
eg.  $2 \rightarrow 2^n$ , or  $-2 \rightarrow (-2)^n$

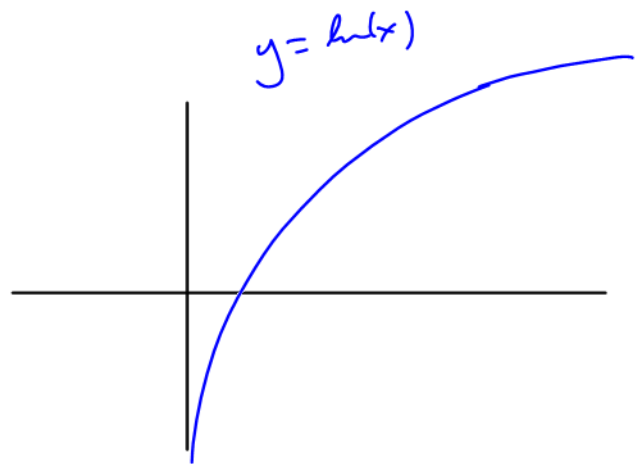
2 major cases  $\rightarrow$   $|r| > 1$ , then  $r^n \rightarrow \infty$   
 $\Rightarrow \frac{a(1-r^n)}{1-r} \boxed{\text{diverges}}$

$|r| < 1$  eg  $r = 1/2, -1/2$

then  $r^n \rightarrow 0$   
 $\Rightarrow S = \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \boxed{\frac{a}{1-r}}$

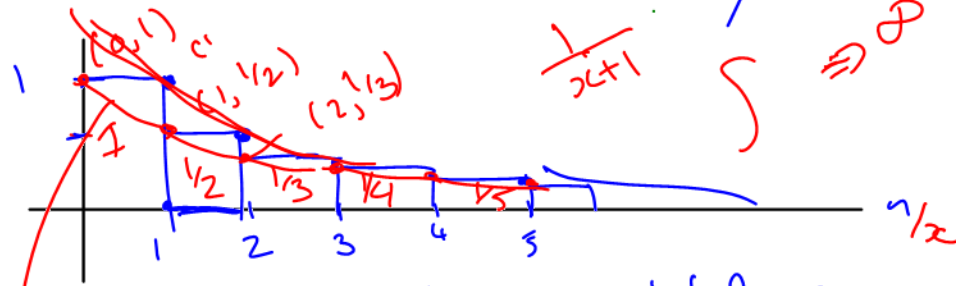
case  $r = 1$  must  
 look at case by case.  
 $+1 -1 +1 -1$  earlier  
 is an example.

single finite value,  
 series converges.



Example 1.4. Does the harmonic series  $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$  converge?

Consider graphical approach; plot terms against  $n$  / Sum of  $n$  terms =



So total rect area = total sum

graph of  $\frac{1}{x+1}$

Area  $\int \frac{1}{x+1} dx$  find w integrals!

as  $n \rightarrow \infty$  area will be some  $\infty$  long.

$$\int_0^n \frac{1}{x+1} dx = \ln|x+1| \Big|_0^n = \ln(n+1) - \ln(1) = \ln(n+1)$$

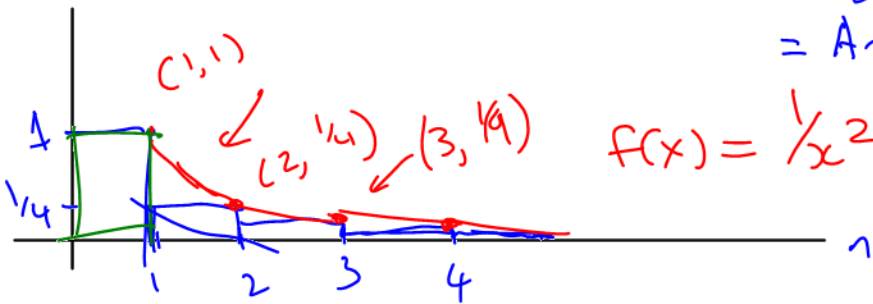
Area  $\int \frac{1}{x} dx = \ln|x|$   
"area  $> \infty$ "  
can it be finite: No!

$\sum_{n=1}^{\infty} \frac{1}{n}$  diverges

Harmonic Series some partial sums:

$s_{10} = 2.92$
$s_{100} = 5.17$
$s_{1000} = 7.48$
$s_{10,000} = 9.87$
$s_{100,000} = 12.09$
$s_{1,000,000} = 14.39$
$s_{10,000,000} = 16.70$
$s_{100,000,000} = 19.00$

Example 1.5 What's the story for the series  $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$ ?



$f(x) = \frac{1}{x^2}$

"Euler" Series some partial sums:

$s_{10} = 1.54977$
$s_{100} = 1.63498$
$s_{1000} = 1.64393$
$s_{10,000} = 1.64483$
$s_{100,000} = 1.64492$
$s_{1,000,000} = 1.64493$
$s_{10,000,000} = 1.64493$

$1 + \text{Area}$

As  $n \rightarrow \infty$  total area  $\rightarrow 2$

$1 < A < 2$

$$1 + \int_1^n \frac{1}{x^2} dx = 1 + (-x^{-1}) \Big|_1^n = 1 + \left(\frac{-1}{n}\right) + \frac{1}{1} = 2 - \frac{1}{n}$$

Area  $\int x^{-2} dx = -x^{-1} = -\frac{1}{x}$   $< 2$

Is this area finite? Yes!

(value??) ick! but we at least know it converges

later

Example 1.6. Find the sum of the arithmetic-geometric series:

$$\sum_{n=1}^{\infty} nr^n = r + 2r^2 + 3r^3 + 4r^4 + \dots$$

$r$					
$r^2$	$r^2$				
$r^3$	$r^3$	$r^3$			
$r^4$	$r^4$	$r^4$	$r^4$		
$r^5$	$r^5$	$r^5$	$r^5$	$r^5$	
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	
					sum of all entries

The arithmetic-geometric series:

$$r + 2r^2 + 3r^3 + 4r^4 + \dots = \frac{r}{(1-r)^2}$$

## 2. Alternating series. *+ Geom are the two big series*

Recall that

$$s_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

is called the  $n$ th partial sum of the series

$$S = \sum_{k=1}^{\infty} a_k$$

We say that  $S$  converges to the limit  $s$  if the partial sums  $s_n$  converge to  $s$  as  $n$  approaches infinity.

A series is alternating if the terms alternate in sign.

For example, the geometric series:  
*(+) (-) (+) (-) (+) (-)*

$s_2 = \frac{1}{3}$

$$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$$

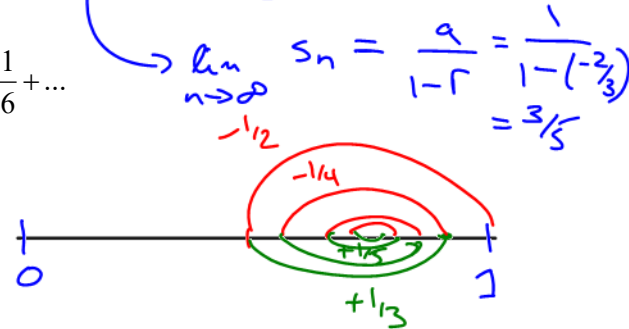
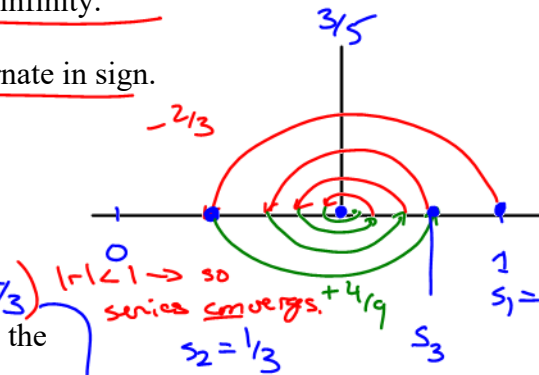
is alternating and converges to  $3/5$ .

Example 2.1. An important example is the alternating harmonic series:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

Does it converge or diverge?

*Does converge!*



*|terms| keep getting smaller and terms  $\rightarrow 0$  as well*

**Convergence test for alternating series.**

Consider the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + \dots \quad (b_n > 0)$$

If

- (i) the  $b_n$  decrease with  $n$
- (ii) and approach zero  $\left( \lim_{n \rightarrow \infty} b_n = 0 \right)$

then the series converges.

*Open question: what value does it converge to? No simple formula.*

and we have no easy formula for the infinite sum.

finite sum approxn.

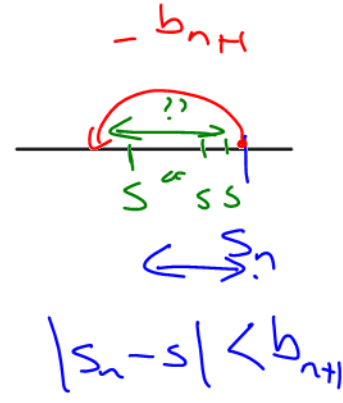
If a series converges, then its partial sums  $s_n$  converge to its sum  $s$ . Thus, by taking  $s_n$  for a large enough  $n$ , we should get a good estimate of  $s$ . The trouble is, we need to know how big to take  $n$  (i.e. how many terms to take) to obtain a desired measure of closeness.

exact / infinite sum

**Error estimation for alternating series.**

Suppose that an alternating series  $\sum_{n=1}^{\infty} (-1)^{n+1} b_n$  ( $b_n > 0$ ) satisfies (i) and (ii) of the convergence test. Then the "error" in approximating its sum  $s$  with the  $n$ th partial sum  $s_n$  is less than the absolute value of the first omitted term:

$$|s - s_n| \leq b_{n+1}$$



Example 2.2: How many terms of the alternating harmonic series:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$

do we need to take to get within  $1/100^{\text{th}}$  of the sum?

$\Rightarrow$  error  $< 0.01$  or  $< \frac{1}{100}$   
 "  $b_{n+1}$  or magnitude of 1<sup>st</sup> term we don't include in the sum  
 Need  $\frac{1}{n+1} < \frac{1}{100}$   
 $100 < n+1$   
 $n > 99$

within 0.01 magnitude of  $n^{\text{th}}$  term is  $\frac{1}{n}$

error in estimate of  $s$  if we use  $s_n$  as the finite approxn

Example 2.3. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}$  correct to three decimal places (i.e., with error less than 0.0005).

Add up first 100 terms. That  $s_n$  will be within 0.01 of exact total.

Guess & check

$n=2, b_3 = \frac{1}{3!} \cong 0.167$  are  $\frac{(-1)^{n-1}}{n!}$

$n=3, b_4 = 0.04$  - magnitudes  $b_n = \frac{1}{n!}$

$n=4, b_5 = \frac{1}{5!} = 0.008$

$n=5, b_6 = \frac{1}{6!} = 0.0014$

$n=6, b_7 = \frac{1}{7!} = 0.00019 < 0.0005$

so if we add up first 6 terms, we get the correct total within 0.0005

Need  $< 0.0005$  I can't solve for  $n$  w/ factorial.

Til now:

eg  $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} ar^{n-1}$$

numbers only!

### 3. Power Series

A power series is an infinite series of the form:

*calculable*  

$$c_0 + c_1x + c_2x^2 + \dots = \sum_{n=0}^{\infty} c_n x^n \quad \text{or} \quad \sum_{n=0}^{\infty} c_n (x-a)^n$$

$\sin(x)$  - not always calculable.  
 these sums are also functions!

*infinite poly's* whose terms are ascending powers of  $x$  or more generally of  $(x-a)$ . When such a series converges, its sum will be a function of  $x$ . The question we will be asking here is the reverse—given a function  $f(x)$ , can we find a power series that converges to it in some  $x$ -interval?

For example, the function  $f(x) = \frac{1}{1-x}$  is recognized as the sum of a geometric series and that gives us a power series expansion

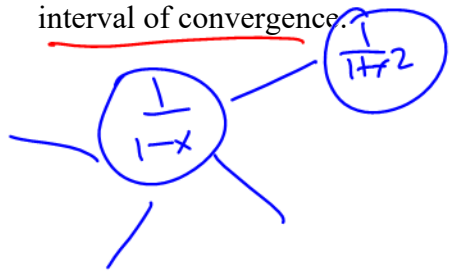
*closed form*  $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$   
*power series form*

like  $\frac{a}{1-r} \rightarrow a=1, r=x$   
 need  $|r| < 1$  or  $|x| < 1$

that converges for  $|x| < 1$ . The *interval of convergence* of a power series is the set of  $x$  for which the series converges.

The interval of convergence is always centered at  $x=a$ —except that we may or may not have convergence at the endpoints. The radius of the interval is called the *radius of convergence* of the series.

*Example 3.1.* Express  $\frac{1}{1+x^2}$  as the sum of a power series in  $x$  and find the interval of convergence.



*substitution  $x \rightarrow (-x^2)$*   

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad \text{for } |x| < 1$$
  

$$\frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \dots \quad \text{for } |-x^2| < 1$$
  
 " 
$$= 1 - x^2 + x^4 - x^6 + x^8 - \dots$$
  
 so 
$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$
  
 interval of convergence:  $|x| < 1$  same  $x$ 's as  $|-x^2| < 1$



Example 3.2. Express  $\frac{x^3}{x+2}$  as the sum of a power series in  $x$  and find the interval of convergence.

$\frac{x^3}{x+2}$    
 subst. transf.   
 Know.   
 $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$    
 $x^3 \left( \frac{1}{1-x} \right) = x^3 ( \dots )$    
 $x^3 \left( \frac{1}{1-x} \right) = \frac{1}{2} x^3 ( \dots )$    
 $\frac{x^3}{2-2x} = \frac{1}{2} x^3 ( \dots )$

*Differentiation and integration.*

It turns out that under standard assumptions of continuity, differentiability and integrability, we can get new power series representations out of old by differentiating and integrating. Also it can be shown that, except for the end points (which we will treat as a special case) this process leaves the interval of convergence unchanged.

Example 3.3 Obtain a power series for  $\frac{1}{(1-x)^2}$  by recognizing it as a derivative.

Know  $\frac{d}{dx} (1-x)^{-1} = \frac{1}{(1-x)^2} = (-1)(1-x)^{-2} (-1) = (1-x)^{-2}$  ✓   
 Know  $\frac{d}{dx} \left( \frac{1}{1-x} \right) = 1 + x + x^2 + x^3 + \dots$  diff'te term by term.   
 $\frac{d}{dx} \left( \frac{1}{1-x} \right) = 0 + 1 + 2x + 3x^2 + 4x^3$    
 $\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + 4x^3$    
 $n=0 \quad n=1 \quad n=2 \quad n=3$

$$= \sum_{n=0}^{\infty} (n+1)x^n = \sum_{n=1}^{\infty} nx^{n-1}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad \text{for } |x| < 1$$

$$\times \frac{1}{2} : \quad \frac{1}{2-2x} = \frac{1}{2} (1 + x + x^2 + x^3 + \dots) \quad \text{for } |x| < 1$$

$\downarrow x \rightarrow -\frac{x}{2}$

↳ subs  $x \rightarrow -x/2$

$$\frac{1}{2-2(-\frac{x}{2})} = \frac{1}{2} (1 + (-\frac{x}{2}) + (-\frac{x}{2})^2 + \dots) \quad \text{for } |-\frac{x}{2}| < 1$$

$$= \frac{1}{2+x}$$

power series for  $\frac{x^3}{2+x}$

$x^3$

$$\frac{x^3}{2+x}$$

$$= x^3 \cdot \frac{1}{2} (1 + (-\frac{x}{2}) + (-\frac{x}{2})^2 + \dots)$$

$$= \frac{1}{2} x^3 - \frac{1}{4} x^4 + \frac{1}{8} x^5 - \frac{1}{16} x^6 + \dots$$

$n=0 \quad n=1 \quad n=2 \quad n=3$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{n+1}} x^{n+3}$$

Example 3.4. Find a power series representation for  $\ln(1+x)$ .

$\frac{1}{1-x}$   
 subs  $x \rightarrow -x$   
 $\frac{1}{1-(-x)} = \frac{1}{1+x}$   
 integrate

relate to  $\frac{1}{1-x}$   
 $= 1 + x + x^2 + x^3 + \dots$   
 subs  $x \rightarrow -x$   
 $= 1 - x + x^2 - x^3 + x^4 - \dots$   
 integrate

$\int \frac{1}{1+x} dx = \ln(1+x) + C = \int (1 - x + x^2 - x^3 + x^4 - \dots) dx$   
 $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$  term by term.

In an equation, if we integrate both sides, the two sides can differ by up to a constant  $\Rightarrow$  just one  $+C$  needed

Find  $+C$  checking at a simple  $x$ :  $x=0$

$LHS = \ln(1+0) + C = RHS = 0$   
 $0 + C = 0 \Rightarrow C = 0$

So  
 $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$   
 $= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$   
 or  $= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

did  $\ln(1+x)$   
 $\frac{1}{1+x} \rightarrow x \rightarrow -x$   
 $\frac{1}{1-x} = 1 + x + x^2 + \dots$

Example 3.5. Find a power series representation for  $\tan^{-1}x$ . or  $\arctan(x)$

Recall  $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2} \rightarrow \int \frac{1}{1+x^2} dx = \arctan(x) + C$

Know  $\frac{1}{1-x} = 1+x+x^2+x^3+\dots$  for  $|x| < 1$

↳ subs  $x \rightarrow (-x^2)$       ↳ subs  $x \rightarrow -x^2$

$\frac{1}{1-(-x^2)} = 1+(-x^2)+(-x^2)^2+(-x^2)^3+\dots$  for  $|x^2| < 1$   
 equivalent  $|x| < 1$

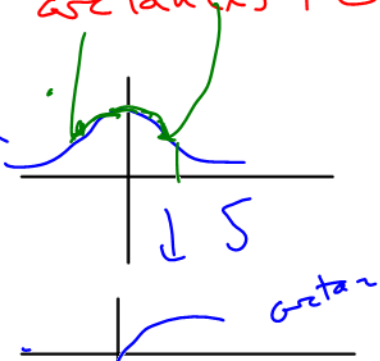
↳ tidy  $\frac{1}{1+x^2} = 1-x^2+x^4-x^6+x^8-\dots$

integrate  $\int dx$  term by term.

$\arctan(x) + C = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

for  $|x| < 1$ .



Example 3.6. Use a power series representation to approximate  $\int_0^{0.5} \frac{1}{1+x^7} dx$  correct to within  $10^{-7}$ .

opt 1) integrate  $\frac{1}{1+x^7}$  using Fond'l Thm of Calculus no simple, non-series anti deriv for  $\frac{1}{1+x^7}$

$\int_0^{0.5} \frac{1}{1+x^7} dx$  correct to



opt 2) estimate integral using LH sum or Trapezoid rule etc.

opt 3) get power series for  $\frac{1}{1+x^7}$ , then integrate series, eval

first n terms

opt 3 looks like.

for  $|x| < 1$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

(subs  $x \rightarrow (-x^7)$ ) ↓ subs  $x \rightarrow (-x^7)$

$$\frac{1}{1-(-x^7)} = 1 + (-x^7) + (-x^7)^2 + (-x^7)^3 + \dots$$

(tidy)

$$= 1 - x^7 + x^{14} - x^{21} + x^{28} - \dots$$

note  $[0, 0.5]$   
is all in  $|x| < 1$

↓  $\int_0^{0.5} \frac{1}{1+x^7} dx$

$$\int_0^{0.5} \frac{1}{1+x^7} dx = x - \frac{x^8}{8} + \frac{x^{15}}{15} - \frac{x^{22}}{22} + \dots$$

$$= \left[ (0.5) - \frac{(0.5)^8}{8} + \frac{(0.5)^{15}}{15} - \frac{(0.5)^{22}}{22} + \dots \right]$$

sub in limits

$$\rightarrow \left[ 0 - \frac{0}{8} + \frac{0}{15} - \dots \right] = 0$$

gag!

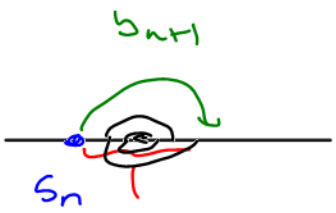
so

$$\int_0^{0.5} \frac{1}{1+x^7} dx = 0.5 - \frac{(0.5)^8}{8} + \frac{(0.5)^{15}}{15} - \frac{(0.5)^{22}}{22} + \dots$$

cannot evaluate the infinite sum

alternating series.

From last week, error in a truncated alternating series is less than the magnitude of the first omitted term



exact value is here somewhere.

Goal: error < first omitted term <  $10^{-7}$

guess & check

$$\frac{(0.5)^{22}}{22} \approx 10^{-8} \checkmark$$

so

$$\int_0^{0.5} \frac{1}{1+x^7} dx \approx 0.5 - \frac{0.5^8}{8} + \frac{(0.5)^{15}}{15}$$

within  $\pm 10^{-7}$

Midterm Tue 6:30 PM.

Rooms: On 6

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Power series  $\rightarrow$  "infinite polynomials"

$$c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$$

Geometric power series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

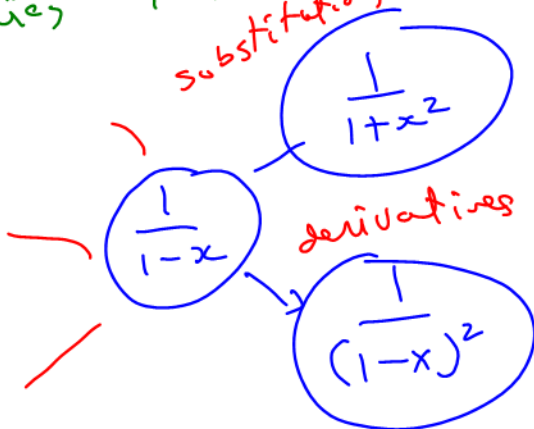
for  $|x| < 1$

Geometric series

$$\frac{a}{1-r}$$

$$= a + ar + ar^2 + ar^3 + \dots \quad |r| < 1$$

substitution



**3.7 General summary of section 3—finding power series.**

The “Go To” example is always the geometric series

$$1 + r + r^2 + \dots = \frac{1}{1-r} \quad (\text{converges for } |r| < 1)$$

1. Use algebra:

$$f(x) = \frac{1}{x+2}$$

2. Use derivative:

$$f(x) = \frac{1}{(1-x)^2}$$

3. Use integral:

$$f(x) = \ln(1+x)$$

4. Substitution or derivative.

$$f(x) = \ln(1+x^2)$$